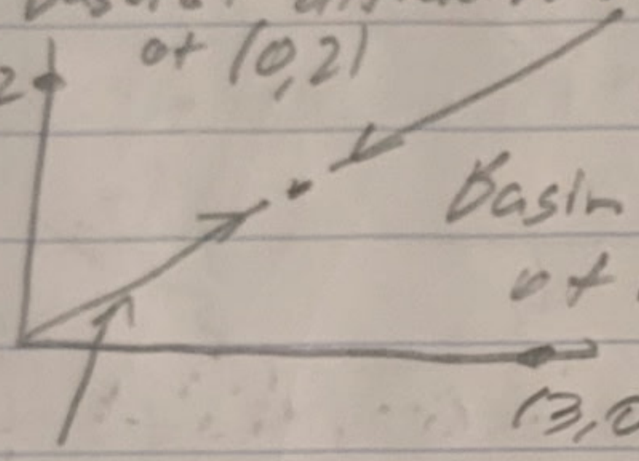
November 5, 2026

$\frac{d}{dt} x(t) = x(3-x-y)$ $\frac{d}{dt} y(t) = y(2-y-x)$

$(x^*, y^*) = (0,0), (1,1), (0,2), (3,0)$

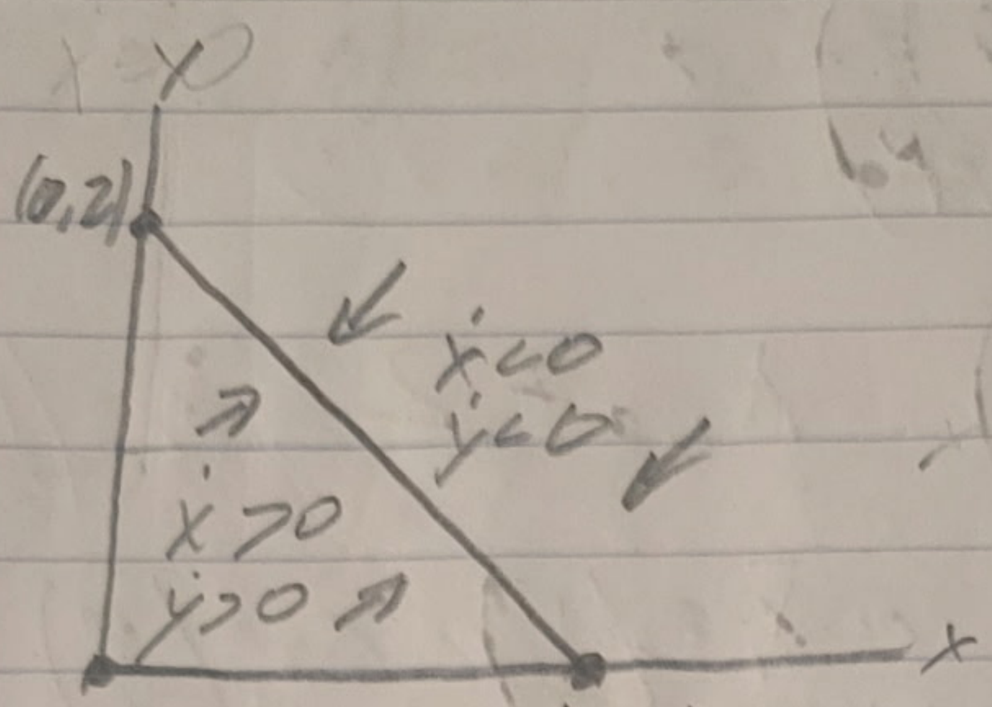


Hetrocyclic Orbit

$\frac{d}{dt} x(t) = x(2-x-y)$ $\frac{d}{dt} y(t) = y(2-x-y)$

$(x^*, y^*) = (0,0), (2,0), (0,2), (1,1)$

There is a line of fixed points: $y = x-2$

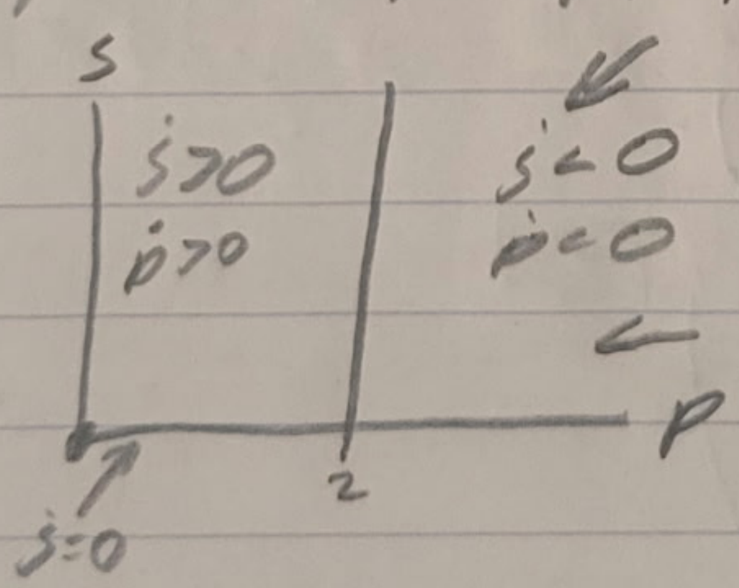


$\dot{x}+\dot{y}=(x+y)(2-x-y)$ $\dot{x}-\dot{y}=(x-y)(2-x-y)$

Let $x(t)+y(t)=P(t)$ and $x(t)-y(t)=S(t)$

$\frac{d}{dt}P(t)=P(t)(2-P(t))$ $\frac{d}{dt}S(t)=S(t)(2-P(t))$

$(P^*, S^*)=(0,0),(2,S)$



$$J(p,s) = \begin{pmatrix} 2-2p & 0 \\ -s & 2-p \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \lambda_{1,2} = 2$$

$$\underline{x}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \underline{x}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$J(2,s) = \begin{pmatrix} -2 & 0 \\ -s & 0 \end{pmatrix}$$

$$\tau = -2 \qquad \Delta = 0, \quad \lambda_{1,2} = \frac{-2 \pm \sqrt{(-2)^2 - 0}}{2}$$

$$= \frac{-2 \pm 2}{2} = -1 \pm 1 = 0, -2$$

$$\frac{d}{dt} x(t) = 1 - 2x - y - xy \qquad \frac{d}{dt} x(t) = y(3x-1)$$

$$(x^*, y^*) = \left(\tfrac{1}{2}, 0\right), \left(\tfrac{1}{3}, \tfrac{1}{2}\right)$$

$$J(x,y) = \begin{pmatrix} -2-y & -1-x \\ 3y & 3x-1 \end{pmatrix}$$

$$J\left(\tfrac{1}{2},0\right)=\begin{pmatrix}-2 & -\frac{3}{2}\\ 0 & \frac{1}{2}\end{pmatrix}\quad \lambda_1=-2\quad \underline{x}_1=\begin{pmatrix}1\\0\end{pmatrix}$$

$$\lambda_2=\tfrac{1}{2}\quad \underline{x}_2=\begin{pmatrix}0\\1\end{pmatrix}$$

$$J\left(\tfrac{1}{3},\tfrac{1}{4}\right)=\begin{pmatrix}-\frac{9}{4} & -\frac{4}{3}\\ \frac{3}{4} & 0\end{pmatrix}$$

$$\lambda_{1,2}=\frac{\tau\pm\sqrt{\tau^2-4\Delta}}{2}=\frac{-\frac{9}{4}\pm\sqrt{\frac{81}{16}-4}}{2}=\frac{-9\pm\sqrt{81-16}}{8}$$

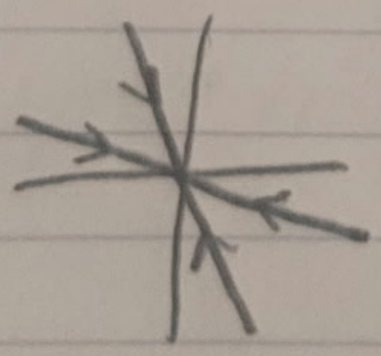
$$=\frac{-9\pm\sqrt{17}}{8}$$

$$\begin{pmatrix}-\frac{9}{4} & -\frac{4}{3}\\ \frac{3}{4} & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\frac{-9\pm\sqrt{17}}{8}\begin{pmatrix}x\\y\end{pmatrix}$$

$$\Rightarrow -\frac{9}{4}x-\frac{4}{3}y=\frac{-9\pm\sqrt{17}}{8}x$$

$$\Rightarrow -\frac{4}{3}y=\frac{9\pm\sqrt{17}}{8}x\Rightarrow -32y=27\pm3\sqrt{17}x$$

$$\underline{x}_1=\begin{pmatrix}-32\\27+3\sqrt{17}\end{pmatrix}=\begin{pmatrix}1\\-0.8\end{pmatrix}\qquad \underline{x}_2=\begin{pmatrix}-32\\27-3\sqrt{17}\end{pmatrix}=\begin{pmatrix}1\\-2.2\end{pmatrix}$$



Conservative System

$$\frac{d}{dt}\underline{x}(t) = f(\underline{x}(t))$$

The system is called conservative if there exists a quantity $E(\underline{x})$ such that $\frac{d}{dt}E(\underline{x}(t)) = 0$ and $E(\underline{x})$ isn't constant.

$$\frac{d}{dt}x(t) = x \qquad \frac{d}{dt}y(t) = -y$$

$$E(x,y) = xy \qquad \frac{d}{dt}E = \frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt}$$

$$= y \cdot x - x \cdot y = 0$$

$\therefore$ The system is conserved

$$m\ddot{x} = F(x) = -\frac{dV}{dx}$$

$$v = \dot{x} \Rightarrow \dot{v} = \ddot{x} = -\frac{1}{m}\frac{dV}{dx}$$

$$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = -\frac{1}{m}\frac{dV}{dx}$$

$$\Rightarrow \ddot{x}\dot{x} = -\frac{1}{m}\frac{dV}{dx}\dot{x} \Rightarrow m\dot{x}\ddot{x} + \frac{dV}{dx}\dot{x} = 0$$

$$\Rightarrow m\dot{x}\ddot{x} = m\frac{d}{dt}\left(\frac{\dot{x}^2}{2}\right), \quad \dot{x}\frac{dV}{dx} = \frac{d}{dt}V(x(t))$$

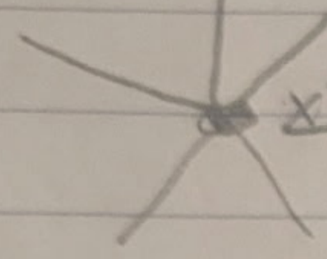
$$\Rightarrow \frac{d}{dt}\left(\frac{m\dot{x}^2}{2}\right) + \frac{d}{dt}V(x(t)) = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{m\dot{x}^2}{2} + V\right) = 0 \Rightarrow \frac{m\dot{x}^2}{2} + V = E(x, \dot{x}) = E(x, v)$$

∴ The system is conserved.

Show that if a system has an attractive fixed point, there is no conserved quantity.

Let system have a conserved quantity and an conserved fixed point.



$E(x^*)$ - value of conserved quantity on a fixed point

Then, the quantity would be constant in the basin of attraction, which contradicts the definition of a conserved quantity.

$$V(x) = -\frac{x^2}{2} + \frac{x^4}{4}$$

Find fixed points, classify their stability, plot phase portrait