-1-

$$\dot{x} = -2\cos x - \cos y$$
$$\dot{y} = -2\cos y - \cos x$$

Show system is reversible.
Find fixed points
plot phase portrait
Show system not conservative

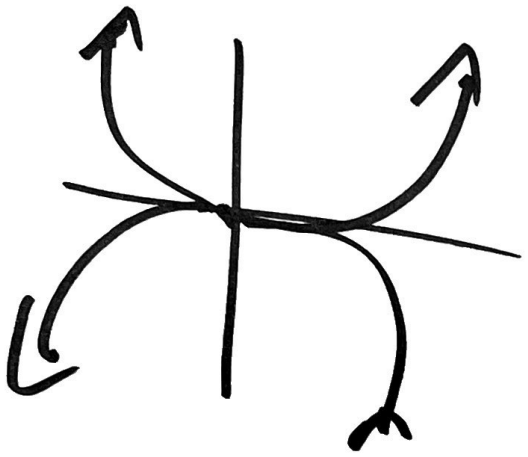
Reversible $t \to -t$
$x \to -x$
$y \to -y$

$$x^* = (2n+1)\pi/2$$
$$y^* = (2m+1)\pi/2$$

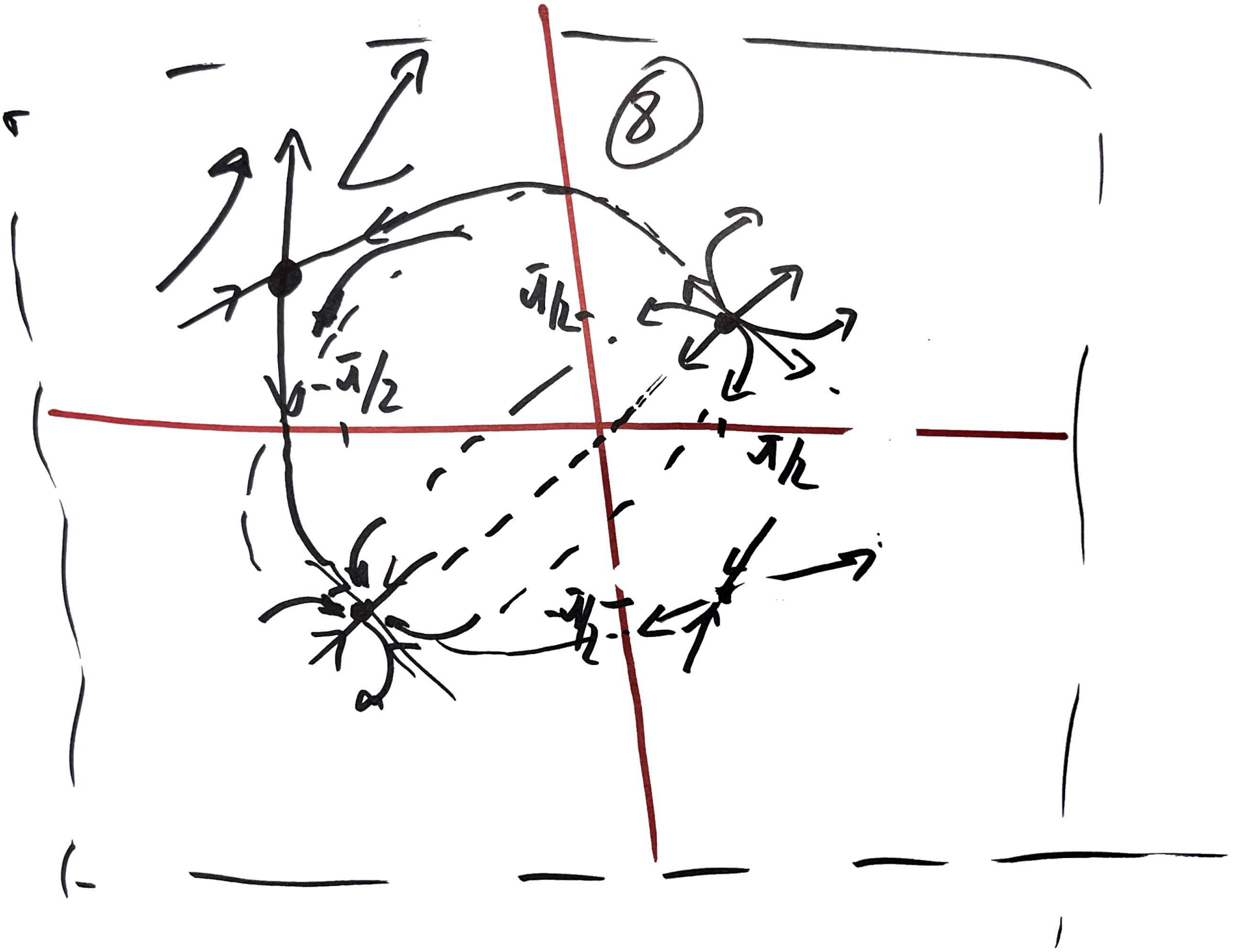
n, m are integers

$$J = \begin{pmatrix} 2\sin x & \sin y \\ \sin x & 2\sin y \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$



$y(t) = C(x(t))^3$


8
$\pi/2$
$-\pi/2$
$\pi/2$
$-\pi/2$

$(\pi/2 \quad \pi/2)$ $J = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$(\pi/2, -\pi/2)$; $J_2 = \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$

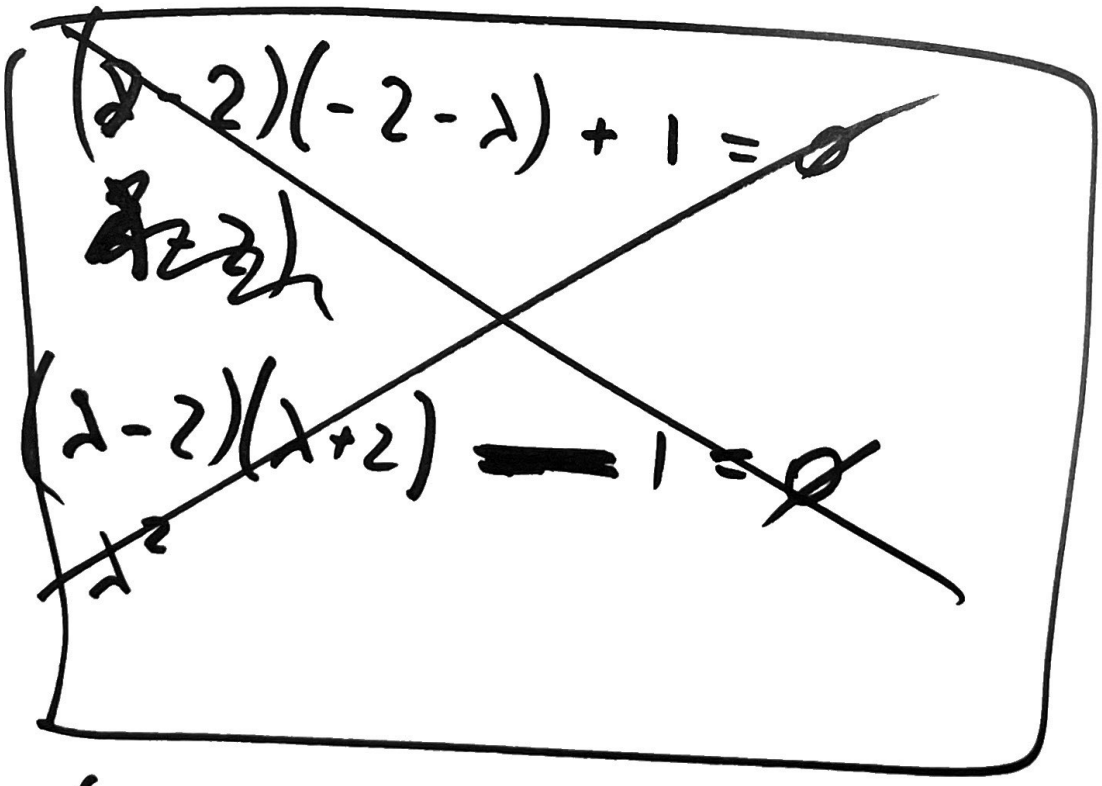
$(-\pi/2, -\pi/2)$ $J_3 = \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$

$(-\pi/2 \quad \pi/2)$ $J_4 = \begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}$

$$J_1 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad \lambda_1 = 1; \quad \underline{x}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_2 = 3; \quad \underline{x}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$J_2 = \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$$



$$(2-\lambda)(-\lambda-2)+1=0$$

$$(\lambda-2)(\lambda+2)+1=0$$

$$\lambda = \pm\sqrt{3}$$

$$\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} = \sqrt{3}\begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix}$$

$$2x_{11} - x_{12} = \sqrt{3}\,x_{11}$$

$$(2-\sqrt{3})\,x_{11} = x_{12}$$

$$\underline{x}_1 = \begin{pmatrix} 1 \\ 2-\sqrt{3} \end{pmatrix}$$

$$\underline{x}_2 = \begin{pmatrix} 1 \\ 2+\sqrt{3} \end{pmatrix}; \quad \lambda_2 = -\sqrt{3}$$

5

- $J_3 = \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$

$Ax = \lambda x$

$-Ax = (-\lambda)x$ $\quad (-\vec{u}_2, -\vec{u}_1)$

$\lambda_1 = -1; \quad x_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\lambda_2 = -3; \quad x_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

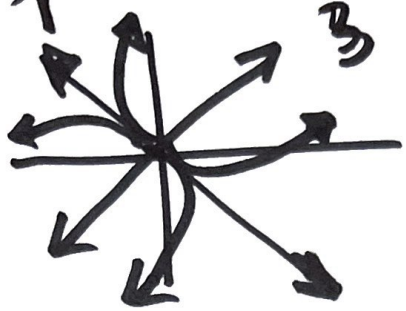
System is not conservative due to attractive fixed point.

- J_4; $\lambda_1 = -\sqrt{3} \quad x_1 = \begin{pmatrix} 1 \\ 2-\sqrt{3} \end{pmatrix}$

$\lambda_2 = \sqrt{3} \quad x_2 = \begin{pmatrix} 1 \\ 2+\sqrt{3} \end{pmatrix}$

-6-

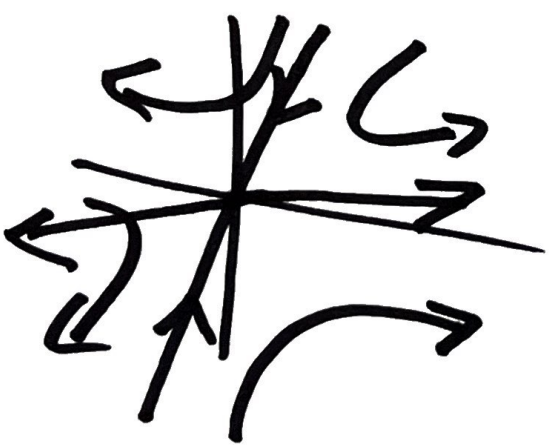
- $\left(\sqrt{3}/2, \sqrt{3}/2\right) = \lambda_1$ $\quad \lambda_1 = 1$, $\lambda_2 = 3$



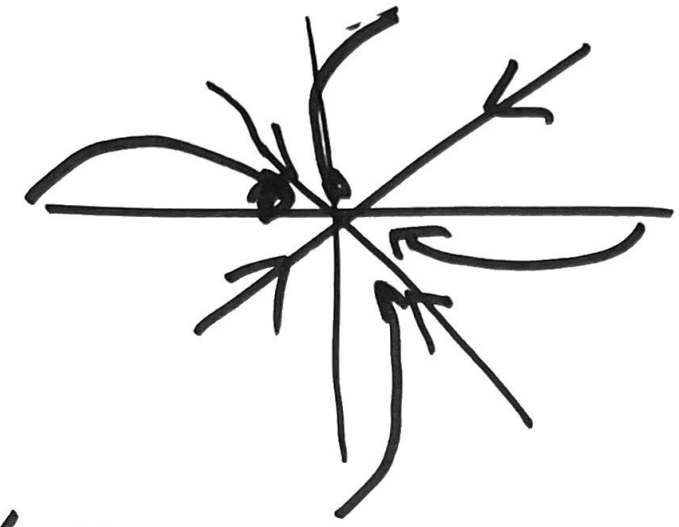
- $\left(\sqrt{3}/2, -\sqrt{3}/2\right)$

$\lambda_1 = \sqrt{3}$ $\quad x_1 = \begin{pmatrix} 1 \\ 2-\sqrt{3} \end{pmatrix}$

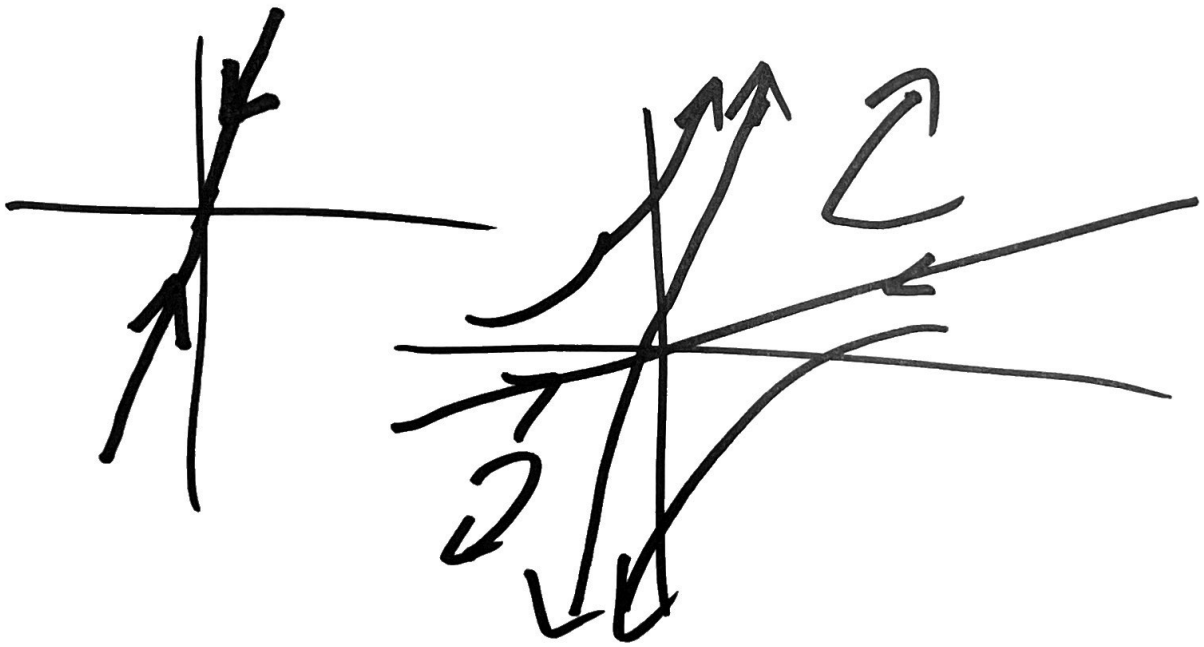
$\lambda_2 = -\sqrt{3}$ $\quad x_2 = \begin{pmatrix} 1 \\ 2+\sqrt{3} \end{pmatrix}$



$(-\sqrt{2}, -\sqrt{2})$



$(-\sqrt{2}, \sqrt{2})$



-10-

$$\ddot{x} + \sin x = 0$$

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\sin x \end{cases}$$

$(0,0)$; $(\pi,0)$

$(n\cdot\pi,0)$; $n = 0, \pm 1, \pm 2, \ldots$

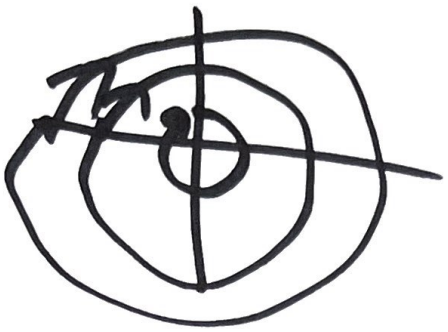
$$J = \begin{pmatrix} 0 & 1 \\ -\cos x & 0 \end{pmatrix}$$

$\underline{x}^* = (0,0)$; $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; $\lambda = \pm i$;

$\underline{x}_1 = \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$

$\underline{x}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\frac{d}{dt}\underline{x} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

11. $(\emptyset, \emptyset)$ is a linear center



$(2n\pi, \emptyset)$ is a linear center

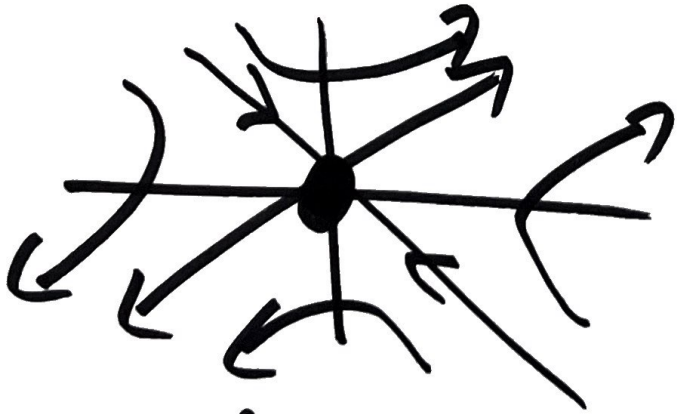
- $(\pi, \emptyset)$

$$J(\pi, \emptyset) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$((2n+1)\pi, \emptyset)$

$\lambda_1 = 1 \; ; \; x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = -1 \quad x_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$



$(\pi, \emptyset)$

12

$$\ddot{x} + \sin x = 0$$

$$\ddot{x} = -\sin x = -\frac{d}{dx} V(x)$$

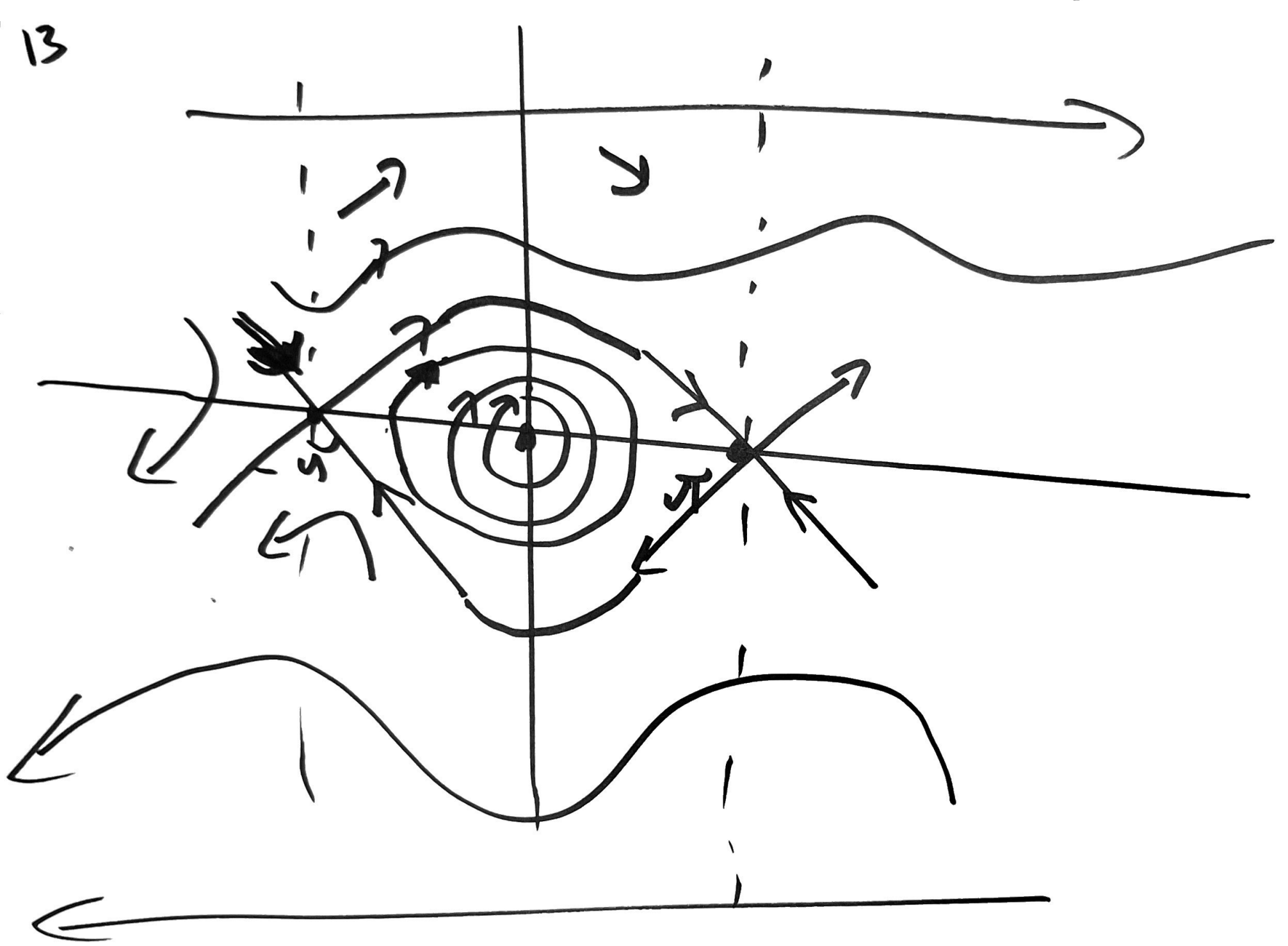
$$V(x) = -\cos(x)$$

$$E(x, \dot{x}) = \dot{x}^2/2 - \cos x$$

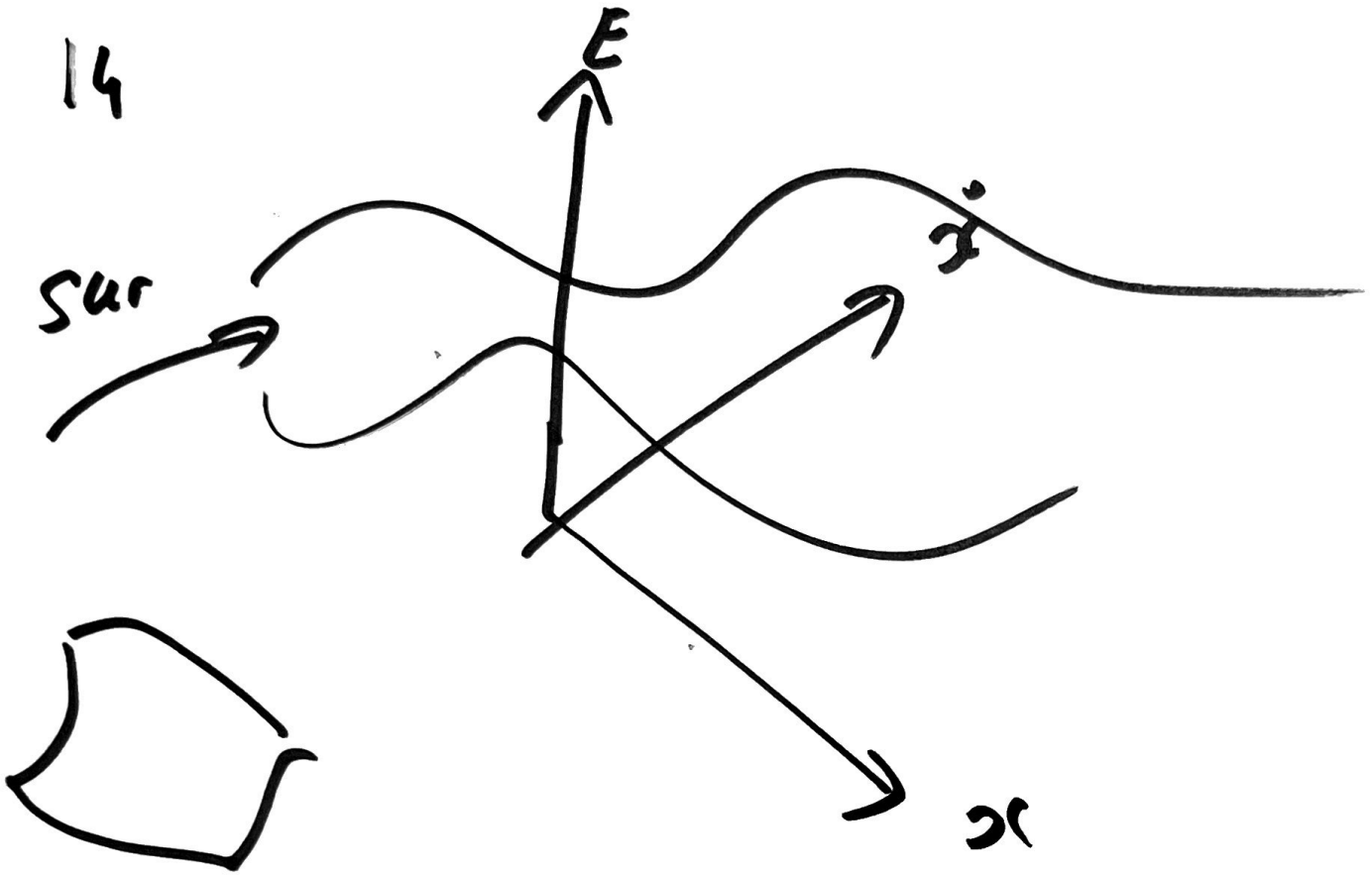
$$\frac{d}{dt} E(x, \dot{x}) = \dot{x}\ddot{x} + \sin x \cdot \dot{x} =$$

$$= \dot{x}(\ddot{x} + \sin x) = 0$$

13



14



-15

$$\ddot{x} + \dot{x} + \sin x = 0$$

$$E = \frac{\dot{x}^2}{2} + \cos x$$

$$\frac{d}{dt} E(t) = -\dot{x}^2$$